MEMHEMATICS Class – 9^{th} and 10 th

1. POLYNOMIALS

Definition of a polynomial in one variable, with examples and counter examples. Coefficients of a polynomial, terms of a polynomial and zero polynomial. Degree of a polynomial. Constant, linear, quadratic and cubic polynomials. Monomials, binomials, trinomials. Factors and multiples. Zeros of a polynomial. Motivate and State the Remainder Theorem with examples. Statement and proof of the Factor Theorem. Factorization of $ax^2 + bx + c, a \neq 0$ where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem.

Recall of algebraic expressions and identities. Verification of identities:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(x \pm y)^3 = x^3 \pm y^3 \pm 3xy (x \pm y)$

$x^3 \pm y^3 = (x \pm y) (x^2 \pm xy + y^2)$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$ and their use in factorization of polynomials.

2. LINEAR EQUATIONS IN TWO VARIABLES

Recall of linear equations in one variable. Introduction to the equation in two variables.

Focus on linear equations of the type ax+by+c=0. Prove that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers, plotting them and showing that they lie on a line. Graph of linear equations in two variables. Examples, problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.

UNIT III: COORDINATE GEOMETRY

1. COORDINATE GEOMETRY

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane.

GEOMETRY

1. **INTRODUCTION TO EUCLID'S GEOMETRY**
2. History - Geometry in India and Euclid's geometry. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem, for example:

- (Axiom) 1. Given two distinct points, there exists one and only one line through them.
- (Theorem) 2. (Prove) Two distinct lines cannot have more than one point in common.

2. LINES AND ANGLES

1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
2. (Prove) If two lines intersect, vertically opposite angles are equal.
3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
4. (Motivate) Lines which are parallel to a given line are parallel.
5. (Prove) The sum of the angles of a triangle is 180°.
6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

3. TRIANGLES

1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle.
5. (Prove) The angles opposite to equal sides of a triangle are equal.
6. (Motivate) The sides opposite to equal angles of a triangle are equal.
7. (Motivate) Triangle inequalities and relation between 'angle and facing side' inequalities in triangles.

4. QUADRILATERALS

1. (Prove) The diagonal divides a parallelogram into two congruent triangles.
2. (Motivate) In a parallelogram opposite sides are equal, and conversely.
3. (Motivate) In a parallelogram opposite angles are equal, and conversely.
4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely.

6. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and (motivate) its converse.

5. AREA

Review concept of area, recall area of a rectangle.

1. (Prove) Parallelograms on the same base and between the same parallels have the same area.
2. (Motivate) Triangles on the same (or equal base) base and between the same parallels are equal in area.

6. CIRCLES

Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter, chord, arc, secant, sector, segment subtended angle.

1. (Prove) Equal chords of a circle subtend equal angles at the center and (motivate) its converse.
2. (Motivate) The perpendicular from the center of a circle to a chord bisects the chord and conversely, the line drawn through the center of a circle to bisect a chord is perpendicular to the chord.
3. (Motivate) There is one and only one circle passing through three given non-collinear points.
4. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the center (or their respective centers) and conversely.
5. (Prove) The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
6. (Motivate) Angles in the same segment of a circle are equal.
7. (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
8. (Motivate) The sum of either of the pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.

7. CONSTRUCTIONS

1. Construction of bisectors of line segments and angles of measure 60°, 90°, 45° etc., equilateral triangles.
2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle.
3. Construction of a triangle of given perimeter and base angles.

UNIT V: MENSURATION

1. AREAS

Area of a triangle using Heron's formula (without proof) and its application in finding the area of a quadrilateral.

2. SURFACE AREAS AND VOLUMES

Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/cones.

UNIT VI: STATISTICS & PROBABILITY

1. STATISTICS

Introduction to Statistics: Collection of data, presentation of data - tabular form, ungrouped / grouped, bar graphs, histograms (with varying base lengths), frequency polygons, qualitative analysis of data to choose the correct form of presentation for the collected data. Mean, median, mode of ungrouped data.

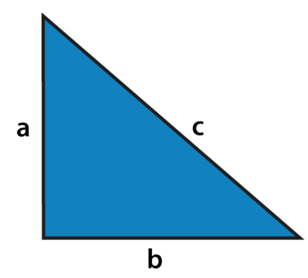
2. PROBABILITY

History, Repeated experiments and observed frequency approach to probability.

Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real - life situations, and from examples used in the chapter on statistics).

Some basic Theorem :

Pythagorean theorem :-



Hypotenuse^2 = Perpendicular^2 + Base^2

$$c^2 = a^2 + b^2$$

The side opposite to the right angle (90°) is

By Pythagoras Theorem –

Area of square A + Area of square B = Area of square C

IMP. : Pythagorean theorem is only applicable to Right-Angled triangle.

the sum of the squares on the legs of a right triangle is equal to the square on the hypotenuse (the side opposite the right angle)—or, in familiar algebraic notation, $a^2 + b^2 = c^2$. Although the theorem has long been associated with Greek mathematician-philosopher Pythagoras

The square of the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.

Pythagoras theorem states that "**In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides**".

Apollonius's theorem

If O be the mid-point of the side MN of the triangle LMN, then $LM^2 + LN^2 = 2(LO^2 + MO^2)$.

Appolonius' theorem relates the length of median with the sides

Proof: Let us choose origin of rectangular Cartesian co-ordinates at O and x-axis along the side MN and OY as the y – axis . If MN = 2a then the co-ordinates of M and N are (- a, 0) and (a, 0) respectively. Referred to the chosen axes if the co-ordinates of L be (b, c) then

$LO^2 = (b - 0)^2 + (C - 0)^2$, [Since, co- ordinates of O are (0, 0)]

$= b^2 + c^2$;

$MO^2 = (- a - 0)^2 + (0 – 0)^2 = a^2$

$LM^2 = (b + a)^2 + (c – 0)^2 = (a + b)^2 + c^2$

And $LN^2 = (b - a)^2 + (c - 0)^2 = (a - b)^2 + c^2$

Therefore, $LM^2 + LN^2 = (a + b)^2 + c^2 + (b - a)^2 + c^2$

$= 2(a^2 + b^2) + 2c^2$

$= 2a^2 + 2(b^2 + c^2)$

$= 2MO^2 + 2LO^2$

$= 2(MO^2 + LO^2)$.

$= 2(LO^2 + MO^2)$.

Hence Proved

Theorems of circle

1. **Theorem 1:** Equal chords of a circle subtend equal angles at the center.

2. Theorem 2: This is the converse of the previous theorem. It implies that if two chords subtend equal angles at the center, they are equal.

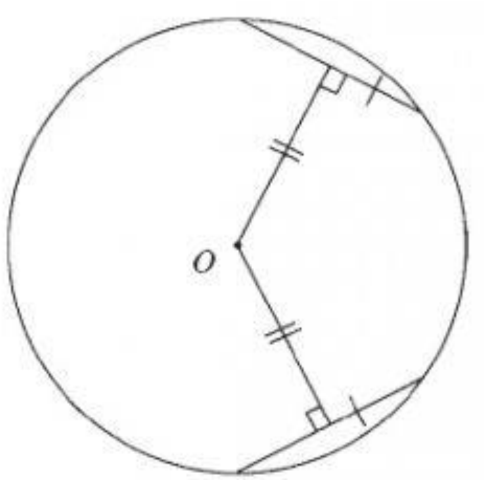
3. Theorem 3: A perpendicular dropped from the center of the circle to a chord bisects it. It means that both the halves of the chords are equal in length.

4. Theorem 4: The line that is drawn through the center of the circle to the midpoint of the chords is perpendicular to it. In other words, any line from the center that bisects a chord is perpendicular to the chord.

5. Theorem 5: If there are three non-collinear points, then there is just one circle that can pass through them.

6. Theorem 6: Equal chords of a circle are equidistant from the center of a circle.

7. Theorem 7: This is the converse of the previous theorem. It states that chords equidistant from the center of a circle are equal in length.



8. Theorem 8: The angle subtended by an arc at the center of a circle is double that of the angle that the arc subtends at any other given point on the circle.

9. Theorem 9: Angles formed in the same segment of a circle are always equal in measure.

10. Theorem 10: If the line segment joining any two points subtends equal angles at two other points that are on the same side, they are concyclic. This means that they all lie on the same circle.

These are some of the basic theorems on the chords and arcs of a circle. We will read about some other theorems in the next chapter. Let us look at some solved examples based on these theorems.

Coordinate Geometry

Coordinate geometry (or analytic geometry) is defined as the study of geometry using the coordinate points. Using coordinate geometry, it is possible to find the distance between two points, dividing lines in m:n ratio, finding the mid-point of a line, calculating the area of a triangle in the cartesian plane, etc. There are certain terms in cartesian geometry that should be properly understood. These terms include:

Co-ordinate Plane?

is one of the branches of geometry where the position of a point is defined using coordinates

Some Solved Question :

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B

Distance between points

$$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$$
$$= \sqrt{1296 + 225} = \sqrt{1521} = 39$$

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Question 7:

Find the point on the x-axis which is equidistant from (2, − 5) and (− 2, 9).

Answer:

We have to find a point on x-axis. Therefore, its y-coordinate will be 0.

Let the point on x-axis be $(x, 0)$.

$$\text{Distance between } (x,0) \text{ and } (2,-5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\text{Distance between } (x,0) \text{ and } (-2,9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$
$$(x-2)^2 + 25 = (x+2)^2 + 81$$
$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$
$$8x = 25 - 81$$
$$8x = -56$$
$$x = -7$$

Therefore, the point is (− 7, 0).

ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

Therefore, ratio between the areas of ΔADE and ΔABC = $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

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$a^2 - b^2 = (a-b)(a+b)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^4 - b^4 = (a-b)(a+b)(a^2 + b^2)$

$a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

Product Formulas

$$(a+b)^2=a^2+2ab+b^2$$

$$(a-b)^2=a^2-2ab+b^2$$

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

$$(a-b)^3=a^3-3a^2b+3ab^2-b^3$$

$$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

$$(a-b)^4=a^4-4a^3b+6a^2b^2-4ab^3+b^4$$

$$(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$$

$$(a+b+c+...)^2=a^2+b^2+c^2+...+2(ab+ac+bc+...)$$

Solid Geometry

Volume of prism = area of base × height

The surface area of a prism is equal to 2 times area of base plus perimeter of base times height.

Surface area of prism = 2 × area of base + perimeter of base × height

Spheres

A sphere is a solid with all its points the same distance from the center.

$\text{SurfaceAreaofaCube}=S=6a^2$
Where, a = Length of the sides of a Cube

$\text{SurfaceAreaofaCylinder}=S=2\pi rh$
$\text{VolumeofaCylinder}=V=\pi r^2 h$
Where, r = Radius of the base of the Cylinder ; h = Height of the Cylinder

$\text{SurfaceAreaofaCone}=S=\pi r(r+\sqrt{h^2+r^2})$
$\text{VolumeofaCone}=V=\pi r^2 h$
Where, r = Radius of the base of the Cone, h = Height of the Cone

$\text{SurfaceAreaofaSphere}=S=4\pi r^2$
$\text{VolumeofaSphere}=V=\frac{4}{3}\pi r^3$
Where, r = Radius of the Sphere